

AD-A049 456

OHIO STATE UNIV COLUMBUS ELECTROSCIENCE LAB
THE CIRCUMFERENTIAL VARIATION OF THE AXIAL COMPONENT OF THE CUR--ETC(U)
OCT 77 P TULYATHAN, E H NEWMAN

F/G 9/5

DAA629-76-G-0067

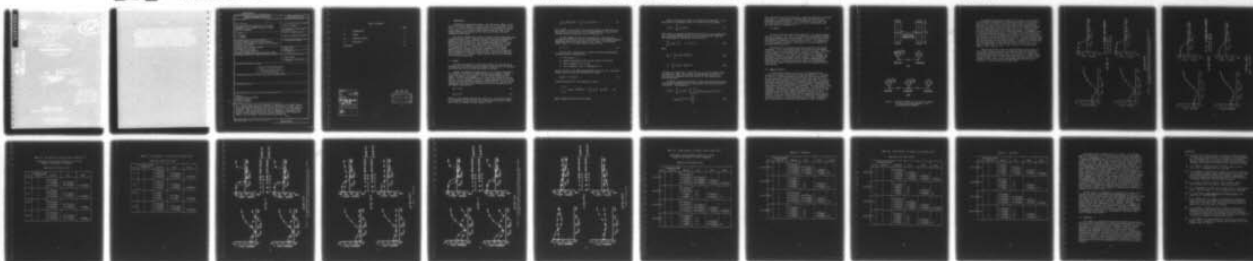
UNCLASSIFIED

ESL-4311-6

ARO-13331.2-EL

NL

| OF |
AD
A049 456



END
DATE
FILMED

3 - 78

DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A049456

AD No. _____
COPY FILE COPY

THE CIRCUMFERENTIAL VARIATION OF THE AXIAL COMPONENT
OF THE CURRENT IS CLOSELY RELATED WITH SOME INTERNAL

Technical report

Electronics Laboratory

Department of Electrical Engineering
University of Illinois - Urbana

ESL-434-6

11

26p

D D C
RECEIVED
FEB 6 1960
RECEIVED
D

Department of the Army
U.S. Army Research Office
Research Triangle Park, North Carolina 27709

UNCLASSIFIED DOCUMENT &
APPROVED FOR PUBLIC RELEASE
EXCEPT WHERE SHOWN
OTHERWISE

402-251

NOTES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government hereby disclaims any responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be recorded by implication or otherwise as in any manner endorsing the product or any other service or corporation, or endorsing any claims or assertions in manufacture, use, or sale of any product or service that may in any way be related thereto.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE CIRCUMFERENTIAL VARIATION OF THE AXIAL COMPONENT OF THE CURRENT IN CLOSELY SPACED THIN-WIRE ANTENNAS		5. TYPE OF REPORT & PERIOD COVERED Technical Report
7. AUTHOR(s) P. Tulyathan E. H. Newman		6. PERFORMING ORG. REPORT NUMBER ESL 4311-6 ✓
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering Columbus, Ohio 43212		8. CONTRACT OR GRANT NUMBER(s) Grant No. DAAG29-76-G-0067 ✓
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army U.S. Army Research Office Research Triangle Park, North Carolina 27709		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE October 1977
		13. NUMBER OF PAGES 22
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
<div style="border: 1px solid black; padding: 5px; text-align: center;"> DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited </div>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Antenna Circumferential variation Method of moments Proximity effects		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
Closely spaced thin-wire antennas are analyzed by the moment method using the piecewise-sinusoidal function to describe the current variation along the length of the wire and a Fourier series for the circumferential variation. Data are presented for the circumferential variation of the surface current density of parallel dipoles obtained by the "thin-wire" theory, wire-grid model, and the present formulation.		

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

	Page
I INTRODUCTION	1
II THEORY	1
III NUMERICAL RESULTS	4
IV CONCLUSION	21
REFERENCES	22

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
Per Doc Form 50	
BY on file	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. AND/OR SPECIAL
A	

DDC
RECEIVED
FEB 2 1978
D

I. INTRODUCTION

A fundamental approximation made in the "thin-wire" theory is that the surface current density is uniform around the circumference of the wires. In order to make this approximation, two conditions are imposed on the wires. First, the wire radius must be much less than a free space wavelength, and second, no two wires pass within a few wire diameters of each other.

This report considers the problem of closely spaced three-dimensional electrically thin wires. Smith [1] and Olaofo [2] had considered the problem of closely spaced infinite cylinders, and Smith included a modification for finite wires. The solution presented here is a modification of the piecewise sinusoidal reaction formulation for "thin-wire" structures [3]. A Fourier series is used to represent the circumferential variation of the axial component of the current. Numerical data illustrate the effects of wire radius and separation on the circumferential variation of the current. The data are compared with the results of "thin-wire" theory and also with a wire-grid model of the closely spaced wires.

II. THEORY

The reaction concept was introduced by Rumsey [4], and applied by Richmond [3] to derive the reaction integral equation (RIE) for radiation and scattering from thin-wires and conducting surfaces.

Consider the problem of scattering by a wire structure enclosed by the surface S , and with the impressed electric and magnetic currents $(\underline{J}_i, \underline{M}_i)$ confined in the volume V_1 . Let the ambient medium be free space. The $e^{j\omega t}$ time dependence of all sources and fields will be suppressed. In the presence of the wire, $(\underline{J}_i, \underline{M}_i)$ generate the field $(\underline{E}, \underline{H})$. From the surface-equivalence theorem of Schelkunoff [5], the wire can be replaced by free space if the following surface-current densities are introduced on the surface S

$$\underline{J}_s = \hat{n} \times \underline{H} \quad (1)$$

$$\underline{M}_s = \underline{E} \times \hat{n} \quad (2)$$

where \hat{n} is the outward directed unit vector on S . By defining $(\underline{J}_s, \underline{M}_s)$ as in Equations (1) and (2), the total field inside the wire is zero. The RIE for the unknown currents $(\underline{J}_s, \underline{M}_s)$ is [3]

$$\iint_S (\underline{J}_S \cdot \underline{E}^m - \underline{M}_S \cdot \underline{H}^m) ds + \iiint_V (\underline{J}_i \cdot \underline{E}^m - \underline{M}_i \cdot \underline{H}^m) dv = 0 \quad (3)$$

where $(\underline{E}^m, \underline{H}^m)$ is the field of test sources $(\underline{J}_m, \underline{M}_m)$ located interior to the surface S and radiating in the homogeneous medium. Below we will consider perfectly-conducting wires, and thus $\underline{M}_S = 0$.

Let each segment of the wire structure have a circular cross section and at each point on the surface define a right-handed orthogonal coordinate system with unit vectors $(\hat{n}, \hat{\phi}, \hat{\ell})$ where \hat{n} is the outward normal vector, $\hat{\ell}$ is directed along the wire axis and

$$\hat{\phi} = \hat{\ell} \times \hat{n}. \quad (4)$$

At this point, Equation (3) is usually simplified by making the following thin-wire approximations:

1. wire radius $a \ll \lambda$
2. neglect integrations over the end surfaces of the wire
3. the $\hat{\phi}$ component of \underline{J}_S is zero
4. the $\hat{\ell}$ component of \underline{J}_S is independent of ϕ .

We will use all of the above approximations except for #4. The surface current density on the wire can then be written as

$$\underline{J}_S(\ell, \phi) = \hat{\ell} J_S(\ell, \phi) \quad (5)$$

Inserting Equation (5) into Equation (3) yields

$$- \int_0^L \int_0^{2\pi} J_S(\ell, \phi) (\hat{\ell} \cdot \underline{E}^m) a d\phi d\ell = \iiint_V (\underline{J}_i \cdot \underline{E}^m - \underline{M}_i \cdot \underline{H}^m) dv \quad (6)$$

where L denotes the overall wire length.

Equation (6) will be solved via the method of moments [6]. To do this $J_s(\ell, \phi)$ is expanded in terms of a finite series as follows:

$$J_s(\ell, \phi) = \sum_{n=1}^N J_n G_n(\ell, \phi) \quad (7)$$

where the J_n are unknown coefficients and the $G_n(\ell, \phi)$ are a known basis set. Enforcing Equation (6) for N distinct test sources yields the following system of simultaneous linear equations

$$\sum_{n=1}^N J_n Z_{mn} = V_m \quad n = 1, 2, \dots, N \quad (8)$$

where

$$Z_{mn} = - \iint_n G_n(\ell, \phi) (\hat{\ell} \cdot \underline{E}^m) ds \quad (9a)$$

$$V_m = \iiint_v (\underline{J}_i \cdot \underline{E}^m - \underline{M}_i \cdot \underline{H}^m) dv \quad (9b)$$

and where the integration in Equation (9a) is over the surface of the n -th expansion mode. The accuracy and computational efficiency of the solution is dependent upon the choices for the expansion and testing functions.

In defining expansion functions we choose the piecewise-sinusoidal function [3] to describe the ℓ variation and a Fourier series for the ϕ variation. The expansion is

$$J_s(\ell, \phi) = \sum_{p=1}^P J_p F_p(\ell) + \sum_{k=1}^K \left\{ \sum_{p=1}^P \left[J_{(2k-1)P+p} F_p(\ell) \cos k\phi + J_{2kP+p} F_p(\ell) \sin k\phi \right] \right\} \quad (10)$$

Thus, the $G_n(\ell, \phi)$ are of the form $F_n(\ell)$, $F_n(\ell)\cos\phi$, $F_n(\ell)\sin\phi$, $F_n(\ell)\cos 2\phi$, $F_n(\ell)\sin 2\phi$, \dots $F_n(\ell)\cos K\phi$ and $F_n(\ell)\sin K\phi$ where the $F_n(\ell)$ are the piecewise-sinusoidal V-dipoles. If P V-dipoles are used to describe the ℓ variation and K terms in the Fourier series are retained, then the total number of unknowns will be

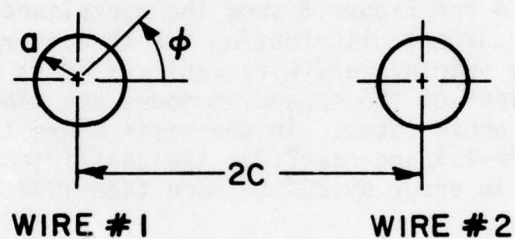
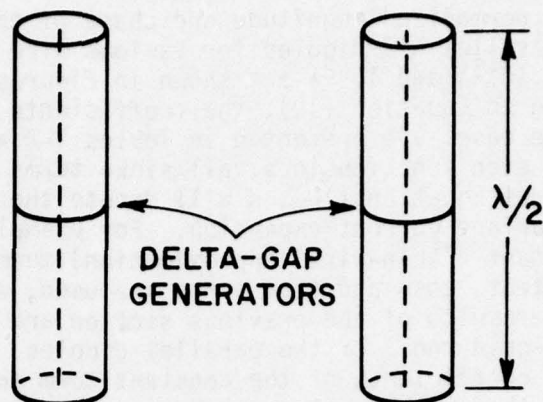
$$N = P(2K+1). \quad (11)$$

The testing functions were chosen identical to the expansion modes. Thus, $\underline{M}_m = 0$ and $\underline{J}_m = \underline{G}_m$. This is an application of Galerkin's method and the impedance matrix will be symmetric. Our initial choice for the test modes was filamentary sources on the wire axis. This choice led to serious problems of relative convergence [7] and was abandoned for the true Galerkin solution.

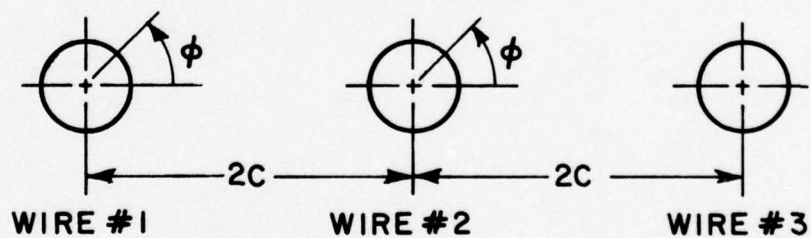
The above choice of expansion and testing functions has several advantages. First, the expansion modes are placed in an overlapping array on the wire so that continuity of current is enforced. Evaluation of the elements in the impedance matrix, Equation (9), requires a quadruple integration, i.e., two to find \underline{E}^m and two to integrate over the surface of the n -th expansion mode. However, since we employ the piecewise-sinusoidal functions, only two of these integrations are done numerically. Advantage can also be taken of the orthogonality of the Fourier modes in evaluating terms where the expansion and test modes overlap.

III. NUMERICAL RESULTS

In this section numerical results will be presented for the surface current density of parallel center-fed dipoles, as illustrated in Figure 1. Results obtained by the method of the previous section will be compared to those of "thin-wire" theory [8], and also to the results of a wire-grid model of the parallel dipoles [9]. In the wire-grid model each thin-wire dipole is modeled by about sixteen extremely thin wires uniformly spaced around its circumference. The dipoles are center-fed by unit voltage delta-gap generators. Although $\underline{J}_s(\ell, \phi)$ varies along the ℓ coordinate, only \underline{J}_s around the center of the wire will be presented. The following calculations demonstrate the ability of the preceding formulation and computer programs to account for the ϕ variation of \underline{J}_s for closely spaced thin and moderately thick parallel dipoles. The surface current densities are normalized with respect to an isolated dipole. In all the computations to follow, 3 modes were used to describe the longitudinal distribution on each $\lambda/2$ dipole.



(a)

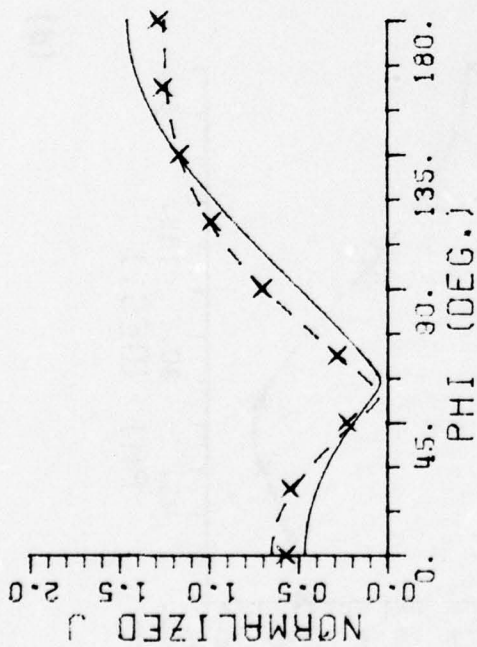


(b)

Figure 1. Parallel dipoles of circular cross section excited by delta-gap generators. The ϕ -coordinates are defined.

The normalized magnitude and phase of the surface current density of two parallel $\lambda/2$ dipoles for various wire spacings, c/a , and wire radii of $10^{-3}\lambda$ and $10^{-2}\lambda$ are shown in Figures 2 and 3, respectively. Referring to Equation (10), the coefficients of the expansion modes for the above cases are presented in Tables 1-A and 1-B. Note that since \underline{J}_s is an even function in ϕ , all $\sin\phi$ terms vanish. Following the notation of Equation (10), K will denote the number of ϕ dependent terms in the surface current expansion. For example, $K=0$ implies that only the constant ("thin-wire" approximation) term is used, $K=1$ implies that the constant, $\cos\phi$ and $\sin\phi$ terms are used, etc. In Figures 2 and 3 note that the results of the previous section are in excellent agreement with the wire-grid model of the parallel dipoles. In Tables 1-A and 1-B note that the coefficients of the constant term for the $K=0$ case is very close to that of the $K=1$ and $K=2$ cases, except in the extreme case where $a = 10^{-2}\lambda$ and $c/a = 1.1$.

Figure 4 and Figure 5 show the normalized magnitude and phase of the surface current distribution for three parallel $\lambda/2$ dipoles for various wire spacings and wire radii of $10^{-3}\lambda$ and $10^{-2}\lambda$, respectively. The coefficient of the expansion modes are tabulated in Tables 2-A and 2-B for the above cases. In the cases where the wires are very close together ($c/a=1.1$ and $c/a=2.0$), the coefficient predicted by the case $K=0$ is in error by 20% to more than 100%.

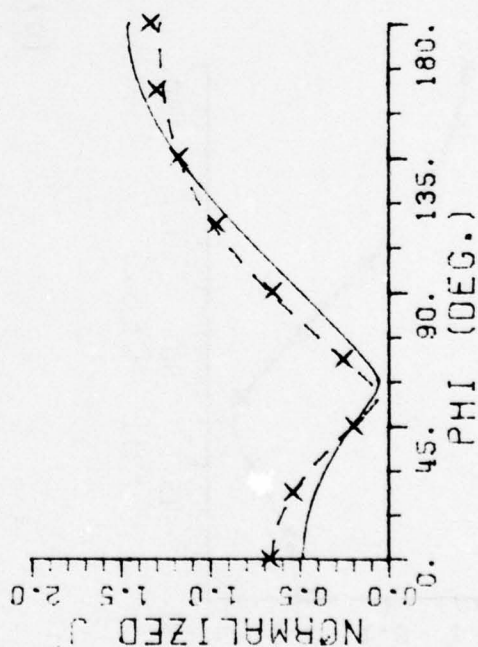
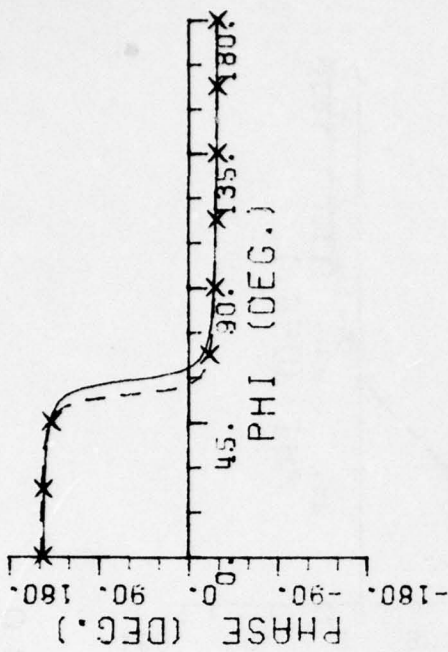


(a) $\%a = 1.1$

X X X WIRE GRID MODEL

— $K = 1$

--- $K = 2$



(b) $\%a = 2.0$

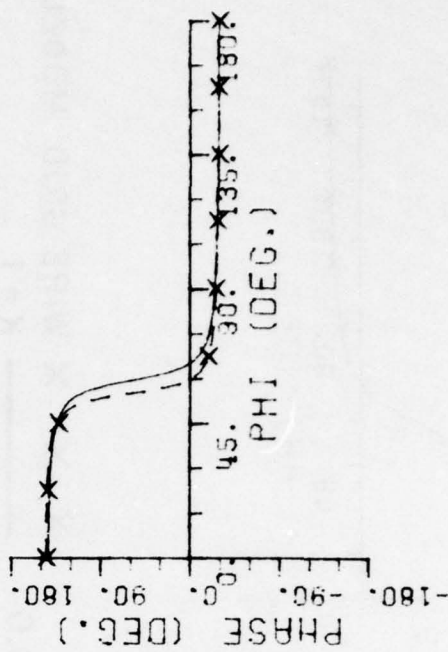
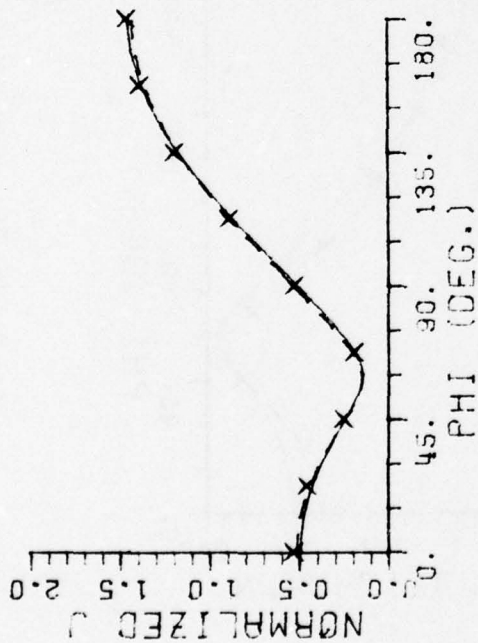
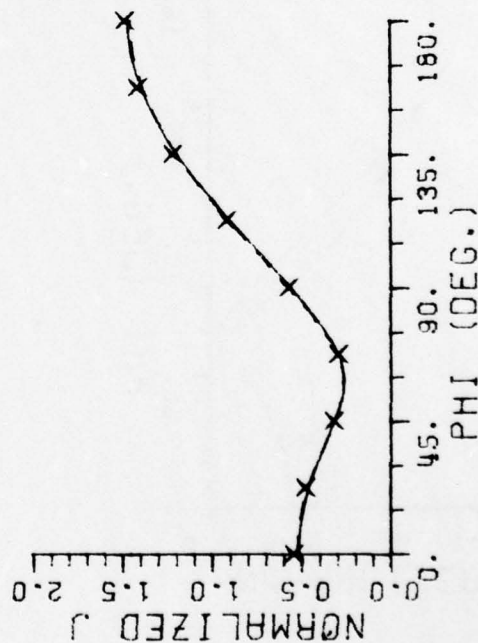
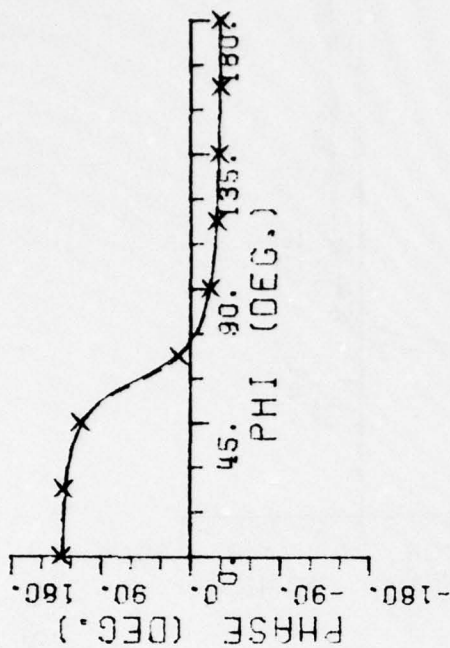


Figure 2. The normalized surface current distribution and phase on two $\lambda/2$ -dipoles ($a=10-3\lambda$) with various wire spacings c/a .



(c) $\%a = 20.0$

X X X WIRE GRID MODEL
 — K = 1
 --- K = 2



(d) $\%a = 50.0$

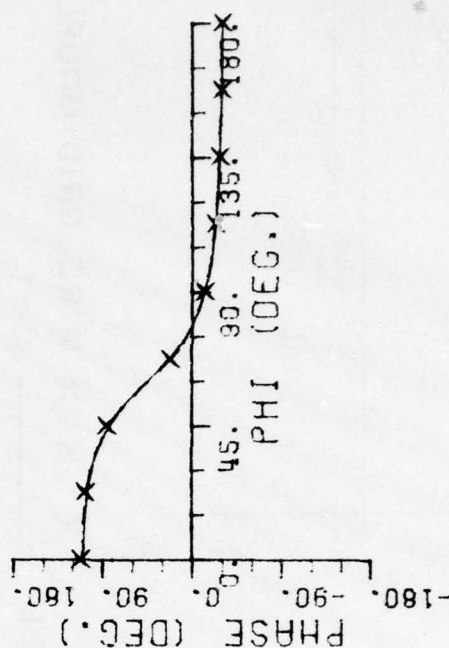
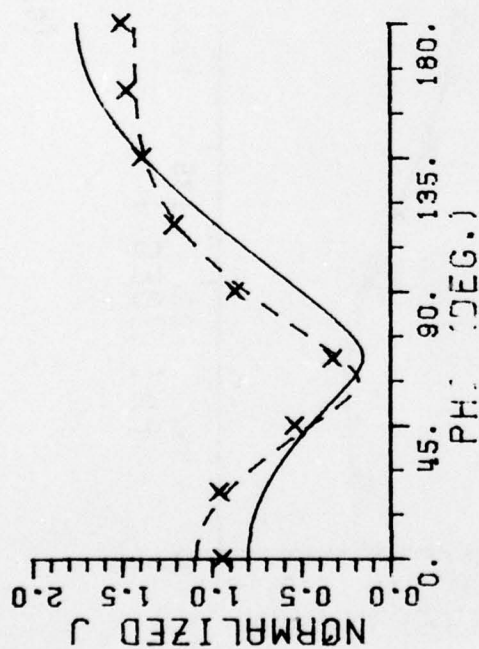
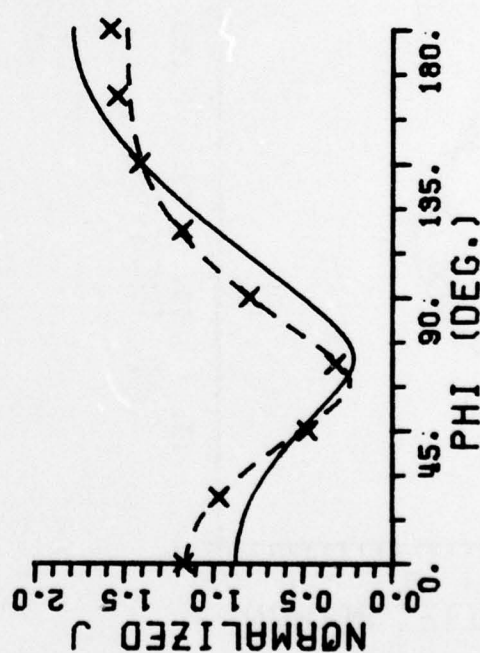
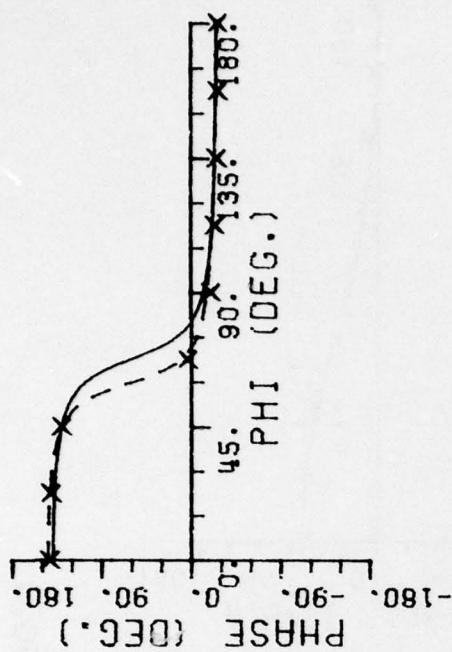


Figure 2. (Continued)



(a) $c/a = 1.1$

X X X WIRE GRID MODEL
 — K = 1
 - - K = 2



(b) $c/a = 2.0$

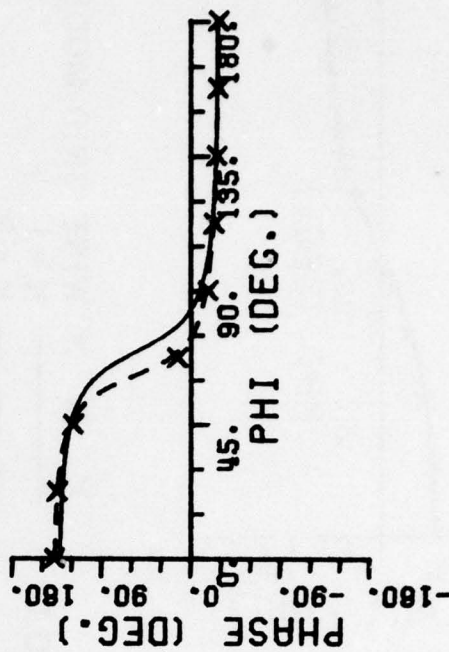
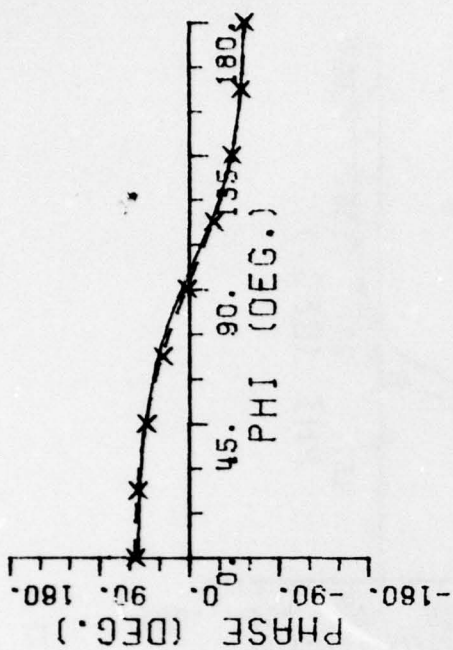
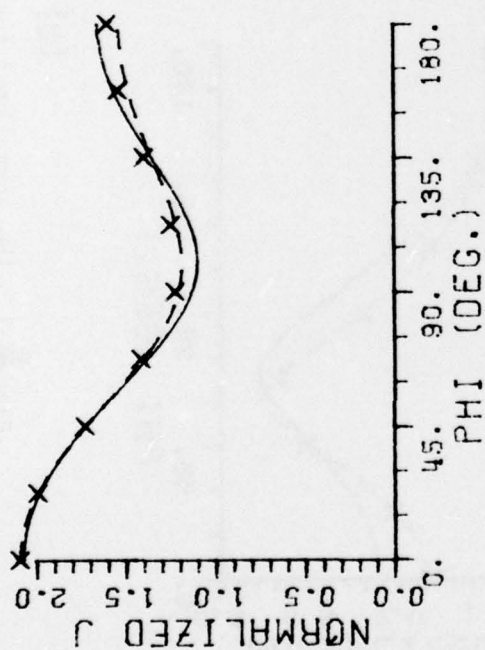
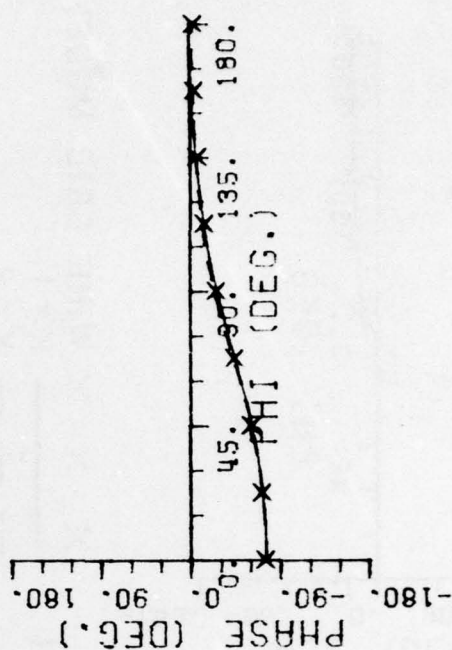
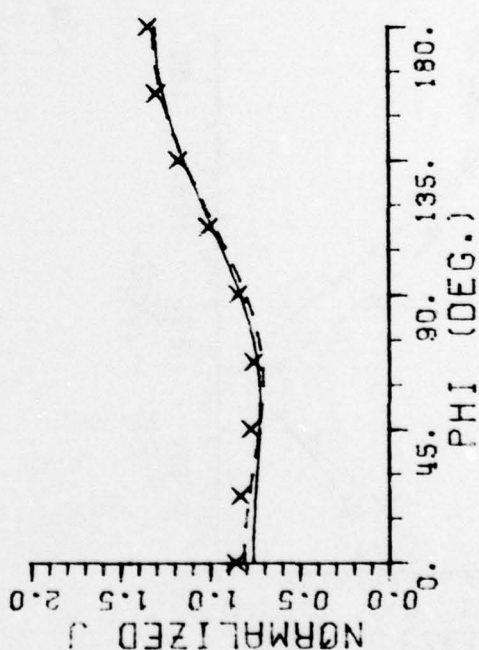


Figure 3. The normalized surface current distribution and phase on two $\lambda/2$ -dipoles ($a=10^{-2}\lambda$) with various wire spacings c/a .



(c) $c/a = 20.0$

X X X WIRE GRID MODEL
 — — — K = 1
 — — — K = 2



(c) $c/a = 50.0$

Figure 3. (Continued)

TABLE 1-A. Two Parallel $\lambda/2$ Dipoles Each of Radius $10^{-3}\lambda$

Coefficients of the Expansion Modes for J_s at the
Center of the Dipole #1 (mA/meter)

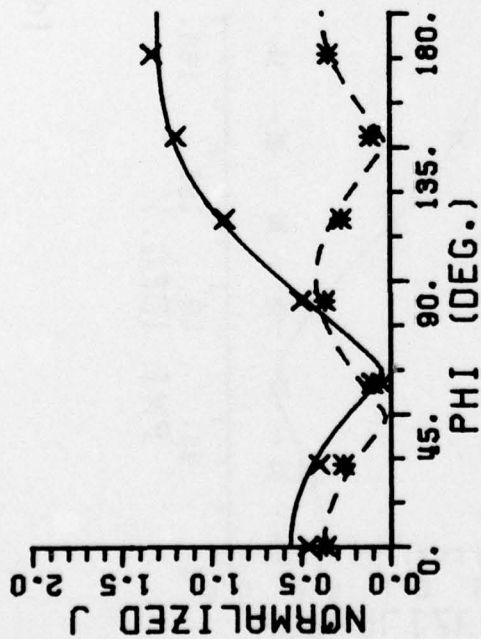
Note that $\sin k\phi$ terms vanish

c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$
1.1	0	86.50/ <u>-26.6</u>	-	-
	1	86.33/ <u>-26.5</u>	166.29/ <u>149.4</u>	-
	2	86.33/ <u>-26.5</u>	164.65/ <u>149.4</u>	34.12/ <u>152.0</u>
2.0	0	86.06/ <u>-26.1</u>	-	-
	1	86.00/ <u>-25.9</u>	168.12/ <u>148.2</u>	-
	2	86.00/ <u>-25.9</u>	167.63/ <u>148.2</u>	30.36/ <u>149.8</u>
20.0	0	88.23/ <u>-20.2</u>	-	-
	1	88.24/ <u>-20.1</u>	166.23/ <u>143.8</u>	-
	2	88.24/ <u>-20.1</u>	166.23/ <u>143.8</u>	4.11/ <u>150.6</u>
50.0	0	97.59/ <u>-13.6</u>	-	-
	1	97.64/ <u>-13.6</u>	164.11/ <u>139.0</u>	-
	2	97.64/ <u>-13.6</u>	164.11/ <u>139.0</u>	1.86/ <u>156.6</u>

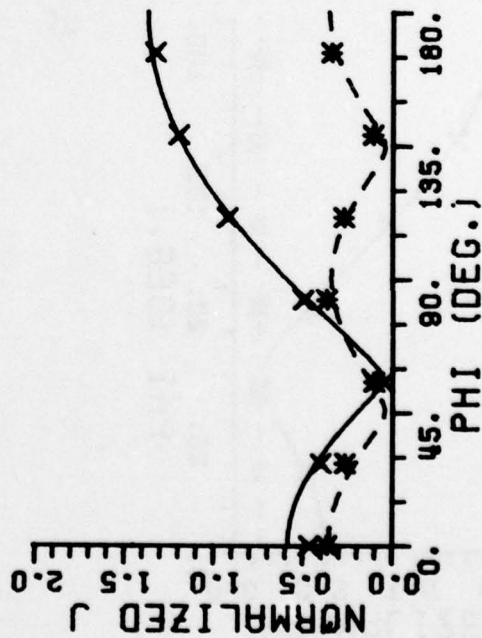
TABLE 1-B. Two Parallel $\lambda/2$ Dipoles Each of Radius $10^{-2}\lambda$

Note that $\sin k\phi$ terms vanish

c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$
1.1	0	7.96/ <u>-15.7</u>	-	-
	1	8.19/ <u>-13.7</u>	20.64/ <u>149.0</u>	-
	2	8.19/ <u>-13.6</u>	20.49/ <u>148.8</u>	5.10/ <u>157.4</u>
2.0	0	8.03/ <u>-11.2</u>	-	-
	1	8.32/ <u>-9.9</u>	21.47/ <u>145.5</u>	-
	2	8.32/ <u>-9.9</u>	21.40/ <u>145.5</u>	4.89/ <u>153.7</u>
20.0	0	18.38/ <u>7.0</u>	-	-
	1	18.35/ <u>6.7</u>	24.51/ <u>83.1</u>	-
	2	18.35/ <u>6.7</u>	24.51/ <u>83.1</u>	1.93/ <u>133.6</u>
50.0	0	13.49/ <u>-26.9</u>	-	-
	1	13.56/ <u>-26.4</u>	10.90/ <u>-146.6</u>	-
	2	13.56/ <u>-26.4</u>	10.90/ <u>-146.6</u>	0.82/ <u>-76.1</u>



(a) $c/a = 1.1$



(b) $c/a = 2.0$

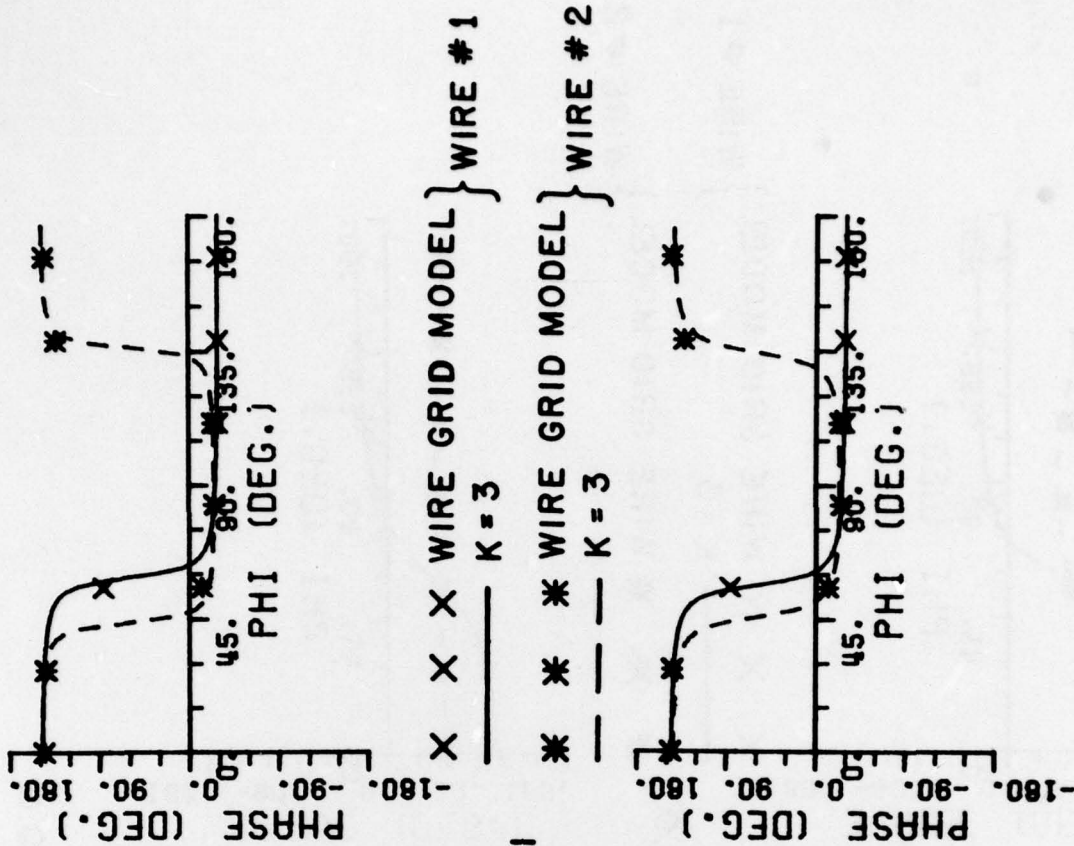
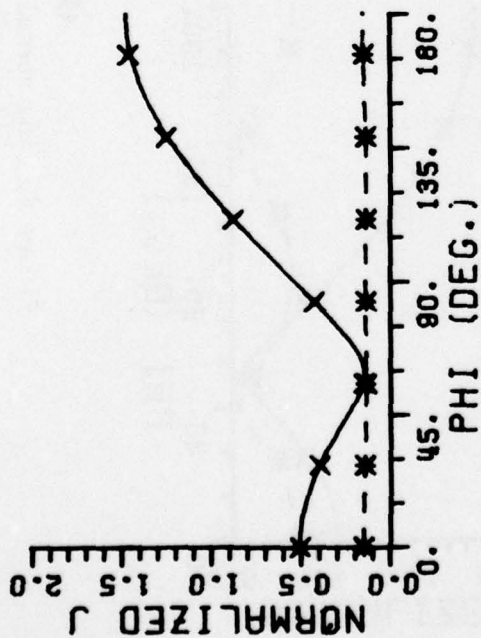
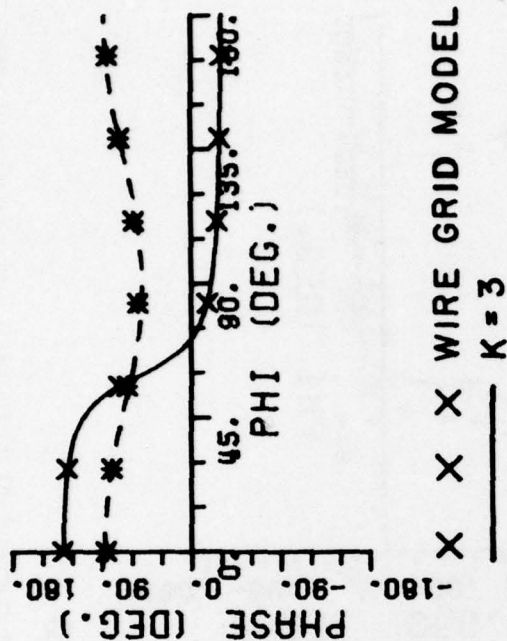


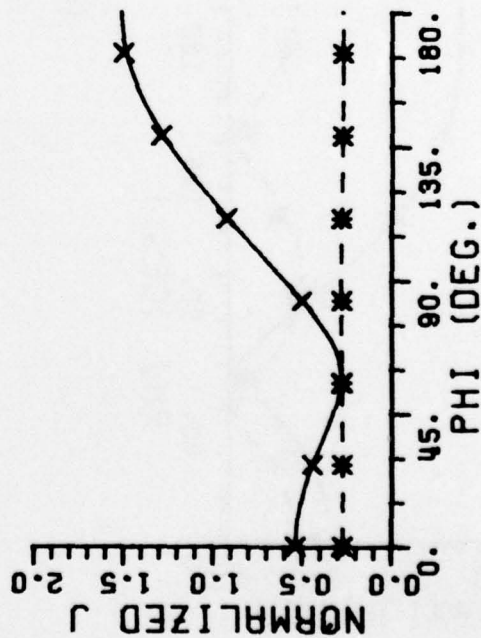
Figure 4. The normalized surface current distribution and phase on three $\lambda/2$ -dipoles ($a=10^{-3}\lambda$) with various wire spacings c/a .



(c) $c/a = 20.0$



X X X WIRE GRID MODEL } WIRE #1
 ——— K = 3
 * * * WIRE GRID MODEL } WIRE #2
 - - - K = 3



(d) $c/a = 50.0$

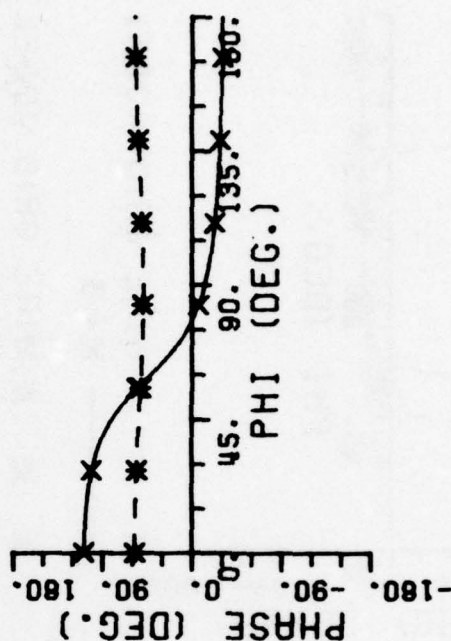
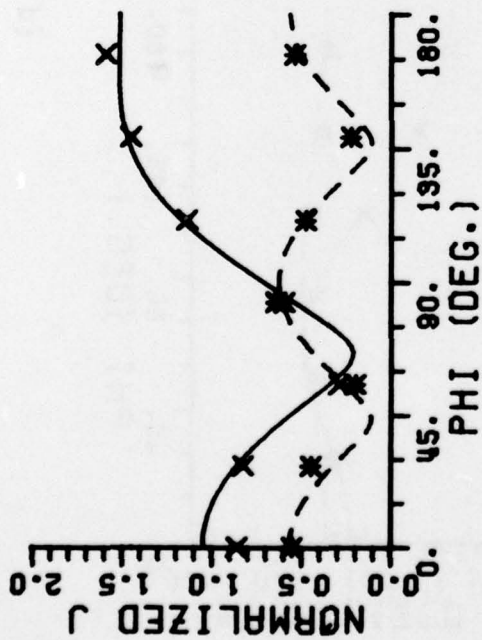
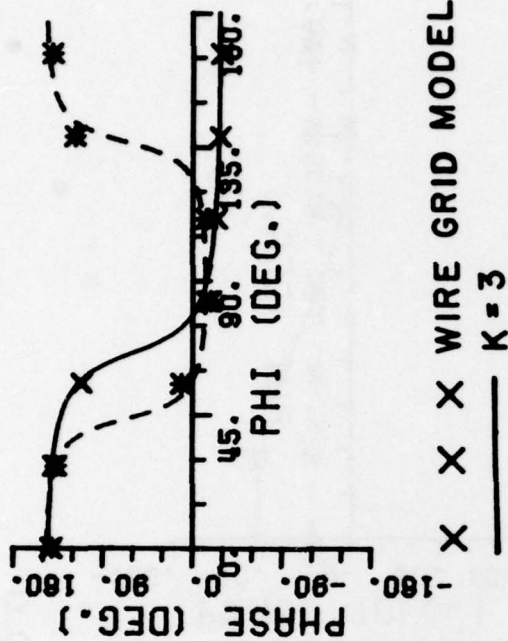
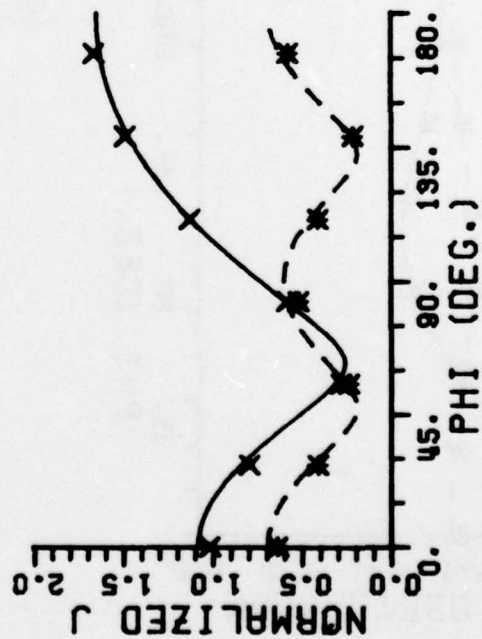


Figure 4. (Continued)

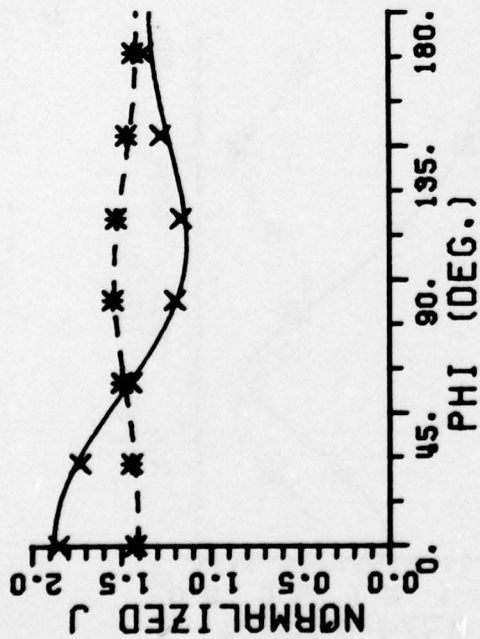


(a) $c/a = 1.1$

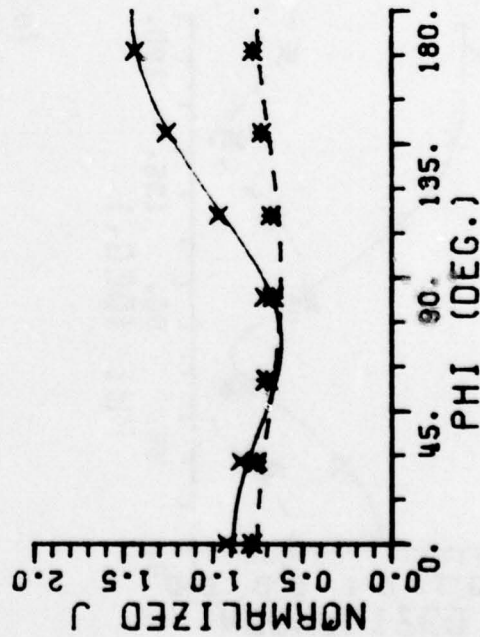
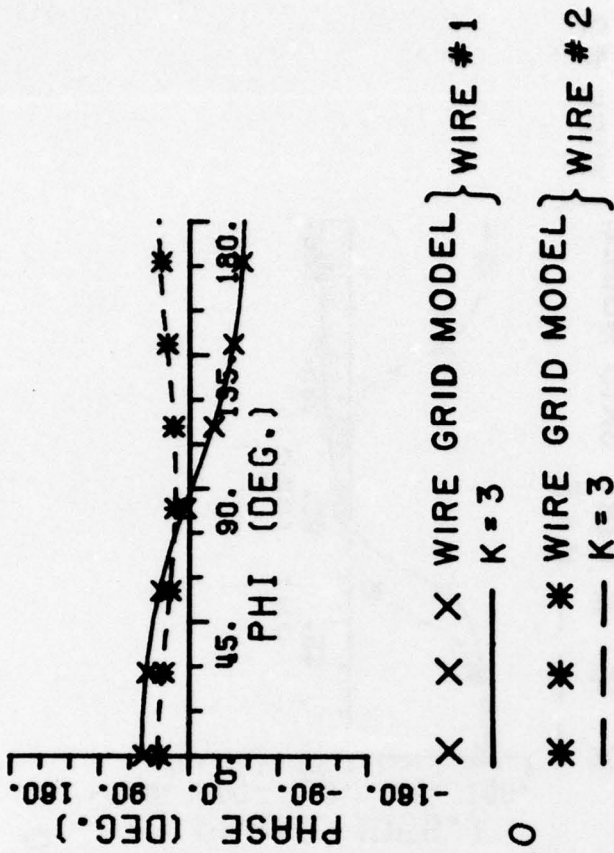


(b) $c/a = 2.0$

Figure 5. The normalized surface current distribution and phase on three $\lambda/2$ -dipoles ($a=10-2\lambda$) with various wire spacings c/a .



(c) $c/a = 20.0$



(d) $c/a = 50.0$

Figure 5. (Continued)

TABLE 2-A. Three Parallel $\lambda/2$ Dipoles Each of Radius $10^{-3}\lambda$

Coefficients of the Expansion Modes for J_s at the
Center of the Dipoles #1 and #2 (mA/meter)

Note that sink ϕ terms vanish

c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$	$\cos 3\phi$
wire #1 { 1.1	0	139.36/ <u>-29.2</u>	-	-	-
	1	79.47/ <u>-27.0</u>	180.71/ <u>149.0</u>	-	-
	2	83.79/ <u>-27.1</u>	169.20/ <u>148.9</u>	21.18/ <u>153.5</u>	-
	3	84.25/ <u>-27.2</u>	168.26/ <u>148.9</u>	20.36/ <u>154.0</u>	6.81/ <u>-38.4</u>
wire #2 { 1.1	0	107.35/ <u>145.5</u>	-	-	-
	1	13.26/ <u>-11.8</u>	0.0	-	-
	2	5.73/ <u>17.0</u>	0.0	66.09/ <u>151.4</u>	-
	3	5.25/ <u>25.2</u>	0.0	68.60/ <u>151.1</u>	0.0
wire #1 { 2.0	0	101.48/ <u>-28.1</u>	-	-	-
	1	85.41/ <u>-27.0</u>	170.37/ <u>147.9</u>	-	-
	2	85.78/ <u>-27.0</u>	168.61/ <u>147.9</u>	17.16/ <u>151.9</u>	-
	3	85.78/ <u>-27.0</u>	168.61/ <u>147.9</u>	17.16/ <u>151.9</u>	1.22/ <u>166.0</u>
wire #2 { 2.0	0	33.54/ <u>136.0</u>	-	-	-
	1	6.39/ <u>63.0</u>	0.0	-	-
	2	6.53/ <u>70.5</u>	0.0	60.49/ <u>149.2</u>	-
	3	6.53/ <u>70.5</u>	0.0	60.46/ <u>149.2</u>	0.0

TABLE 2-A. Continued

c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$	$\cos 3\phi$
wire #1 { 20.0	0	<u>91.89/-21.2</u>	-	-	-
	1	<u>90.07/-20.9</u>	<u>170.14/143.7</u>	-	-
	2	<u>90.75/-20.9</u>	<u>170.14/143.7</u>	<u>2.51/160.0</u>	-
	3	<u>90.75/-20.9</u>	<u>170.14/143.7</u>	<u>2.51/160.0</u>	0.0
wire #2 { 20.0	0	<u>24.35/74.1</u>	-	-	-
	1	<u>23.69/68.7</u>	-	-	-
	2	<u>23.69/68.7</u>	0.0	<u>8.36/150.2</u>	-
	3	<u>23.69/68.7</u>	0.0	<u>8.36/150.2</u>	0.0
wire #1 { 50.0	0	<u>103.08/-13.9</u>	-	-	-
	1	<u>103.67/-13.7</u>	<u>169.30/138.4</u>	-	-
	2	<u>102.67/-13.7</u>	<u>169.30/138.4</u>	<u>1.51/173.1</u>	-
	3	<u>102.67/-13.7</u>	<u>169.30/138.4</u>	<u>1.51/173.1</u>	0.0
wire #2 { 50.0	0	<u>47.96/53.9</u>	-	-	-
	1	<u>47.96/52.7</u>	0.0	-	-
	2	<u>47.96/52.7</u>	0.0	<u>3.86/156.5</u>	-
	3	<u>47.96/53.7</u>	0.0	<u>3.86/156.5</u>	0.0

TABLE 2-B. Three Parallel $\lambda/2$ Dipoles Each of Radius $10^{-2}\lambda$

Note that sink ϕ terms vanish

	c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$	$\cos 3\phi$
wire #1	1.1	0	14.67/ <u>-25.8</u>	-	-	-
		1	7.27/ <u>-12.8</u>	23.57/ <u>148.4</u>	-	-
		2	7.82/ <u>-13.8</u>	22.12/ <u>147.9</u>	3.95/ <u>162.2</u>	-
		3	7.89/ <u>-14.1</u>	21.96/ <u>148.0</u>	3.81/ <u>163.4</u>	1.08/ <u>-51.7</u>
wire #2	1.1	0	14.31/ <u>135.8</u>	-	-	-
		1	2.40/ <u>17.6</u>	0.0	-	-
		2	1.80/ <u>45.1</u>	0.0	9.22/ <u>157.8</u>	-
		3	1.82/ <u>50.2</u>	0.0	9.62/ <u>156.6</u>	0.0
wire #1	2.0	0	10.14/ <u>-17.9</u>	-	-	-
		1	8.52/ <u>-11.8</u>	22.28/ <u>145.3</u>	-	-
		2	8.59/ <u>-11.9</u>	21.96/ <u>145.3</u>	3.42/ <u>163.0</u>	-
		3	8.59/ <u>-11.9</u>	21.96/ <u>145.3</u>	3.42/ <u>163.0</u>	0.34/ <u>-165.8</u>
wire #2	2.0	0	5.97/ <u>113.1</u>	-	-	-
		1	3.11/ <u>74.4</u>	0.0	-	-
		2	3.16/ <u>76.7</u>	0.0	10.10/ <u>152.7</u>	-
		3	3.16/ <u>76.7</u>	0.0	10.10/ <u>152.7</u>	0.0

TABLE 2-B. Continued

c/a	expansion mode used (K)	Constant	$\cos\phi$	$\cos 2\phi$	$\cos 3\phi$
wire #1 { 20.0	0	<u>17.63/2.0</u>	-	-	-
	1	<u>17.76/2.5</u>	<u>20.69/78.2</u>	-	-
	2	<u>17.76/2.5</u>	<u>20.69/78.2</u>	<u>1.91/124.9</u>	-
	3	<u>17.76/2.5</u>	<u>20.69/78.2</u>	<u>1.91/124.9</u>	<u>0.07/159.4</u>
wire #2 { 20.0	0	<u>24.16/21.7</u>	-	-	-
	1	<u>23.83/22.1</u>	0.0	-	-
	2	<u>23.83/22.1</u>	0.0	<u>3.80/129.0</u>	-
	3	<u>23.83/22.1</u>	0.0	<u>3.80/129.0</u>	0.0
wire #1 { 50.0	0	<u>12.40/-28.9</u>	-	-	-
	1	<u>12.50/-28.4</u>	<u>14.54/-149.2</u>	-	-
	2	<u>12.50/-28.4</u>	<u>14.54/-149.2</u>	<u>1.09/-75.6</u>	-
	3	<u>12.50/-28.4</u>	<u>14.54/-149.2</u>	<u>1.09/-75.6</u>	<u>0.03/-8.6</u>
mwire #2 { 50.0	0	<u>11.18/-34.8</u>	-	-	-
	1	<u>11.14/-34.6</u>	0.0	-	-
	2	<u>11.12/-34.6</u>	0.0	<u>1.55/-78.8</u>	-
	3	<u>11.12/-34.6</u>	0.0	<u>1.55/-78.8</u>	0.0

The results shown in Figures 2-5 and in Tables 1 and 2 suggest opposite conclusions as to the accuracy of the thin-wire approximation that J_s is independent of ϕ . First, Figures 2-5 clearly show that J_s is strongly ϕ dependent, even for c/a as large as 50 and a as small as $\lambda/1000$. Thus, one is led to the conclusion that the thin-wire approximation is not valid. Next, Tables 1 and 2 show that, except for extremely small c/a where the thin-wire approximation is known to fail, the ϕ independent term predicted from "thin-wire" theory ($K=0$ case) is essentially identical to the ϕ independent term from the $K=1$ or $K=2$ cases. Since for wires whose radius $a \ll \lambda$ it is the ϕ independent term which dominates the far-zone radiated fields, "thin-wire" theory will predict the proper far-zone fields. Also, if the device used to measure input impedance is sensitive to the average value of voltage/ J_s around the circumference of the wire, then it will also depend only on the ϕ independent modes. Thus, one can be led to the conclusion that the "thin-wire" approximation is in fact valid. This discrepancy can be resolved by stating that the "thin-wire" approximation is not valid for predicting the circumferential distribution of current on a general wire structure, however, it is useful for computing such quantities as impedance and far-zone fields.

At this point it is reasonable to ask the question "why go to all the time and trouble to determine the ϕ dependence of J_s since it ordinarily does not affect either the input impedance or far-zone fields?" The answer is two-fold. First, "thin-wire" computer programs are in such widespread use that any information relative to the basic approximations made in these codes is of value. Secondly, there are quantities which the ϕ dependence of J_s do affect. For example, the conductor loss resistance is a strong function of the circumferential distribution of current on the wire [1]. A knowledge as to how this loss resistance varies with wire radius and wire spacing is important in determining the efficiency of such antennas as the electrically small multiturn loop antenna.

IV. CONCLUSION

This report has considered the problem of determining the circumferential current distribution on closely spaced electrically thin-wires. The solution is a modification of the piecewise sinusoidal reaction formulation for thin-wire structures. A Fourier series is used to represent the ϕ dependence of the wire surface current density. Numerical data presented illustrate that there can be a considerable circumferential variation even when the wire to wire separation exceeds several wire diameters. This can be important in determining antenna efficiency; however, if these wires were to be analyzed by "thin-wire" theory (i.e., neglecting the circumferential variation of the surface current density) the correct impedance and far-zone fields are usually obtained.

REFERENCES

- [1] G. Smith, "The Proximity Effect in Systems of Parallel Conductors and Electrically Small Multiturn Loop Antennas," Technical Report No. 624, December 1971. Division of Engineering and Applied Physics, Harvard University; prepared under Contract N00014-67-A-0298-0005, NR-371-016 for the Joint Services Electronics Program.
- [2] G. O. Olaofe, "Scattering by Two Cylinders," Radio Science, Vol. 5, No. 11, November 1970, pp. 1351-1360.
- [3] J. H. Richmond, "Radiation and Scattering by Thin-Wire Structures in the Complex Frequency Domain," Report 2902-10, July 1973. The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Grant No. NGL 36-008-138 for National Aeronautics and Space Administration.
- [4] V. H. Rumsey, "Reaction Concept in Electromagnetic Theory," Physical Review, Vol. 94, June 15, 1954, pp. 1483-1491.
- [5] S. A. Schelkunoff, "On Diffraction and Radiation of Electromagnetic Waves," Physical Review, Vol. 56, 15 August 1939, pp. 308-316.
- [6] R. F. Harrington, Field Computations by Moment Methods, The MacMillan Co., New York, 1968.
- [7] W. A. Imbriale and P. G. Ingerson, "On Numerical Convergence of Moment Solutions of Moderately Thick Wire Antennas Using Sinusoidal Basis Functions," IEEE Transaction on Antennas and Propagation, Vol. AP-21, May 1973, pp. 363-366.
- [8] J. H. Richmond, "Computer Program for Thin-Wire Structures in a Homogeneous Conducting Medium," Report 2902-12, August 1973, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Grant No. NGL 36-008-138 for National Aeronautics and Space Administration.
- [9] J. H. Richmond, "A Wire-Grid Model for Scattering by Conducting Bodies," IEEE Transactions on Antennas and Propagation, Vol. AP-14, November 1966, pp. 782-786.